Study of Roll Stability Derivatives

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Nomenclature

AR = wing or fin aspect ratio
b = inclusive span
C_{l_p} = roll damping stability derivative
C_{l_δ} = roll due to differential fin cant stability derivative
d = body diameter
E = Eastman's correlation parameter
l_R = reference length
M = Mach number
p = rate of roll
S_F = inclusive area of a pair of fins
S_R = reference area
S_0 = body cross-sectional area
y_c = lateral area center of a fin

Superscript

AD = after Adams and Dugan[2]

Introduction

EASTMAN[1] devised an empirical correlation between the roll damping and the roll due to fin cant stability derivatives for cruciform-tailed missiles:

$$C_{l_p} = -2.15(y_c/d)C_{l_\delta} \tag{1}$$

This correlation is based on experimental data of 10 configurations at Mach numbers between 0 and 4.0. It enables a reliable estimate of the dynamic stability derivative C_{l_p} based on the static stability derivative C_{l_δ}. Eastman[1] referred to the Adams and Dugan[2] analysis of the roll stability derivatives and concluded that the data he used "would not correlate using the correlation curve used by them."

Adams and Dugan's classic report is revisited in an attempt to evaluate how well their analysis, which is based on slender-wing theory, can predict Eastman's empirical correlation.

Analysis

The reference lengths and areas used in Refs. 1 and 2 are summarized in Table 1. According to Table 1, the relationships between the stability derivatives as defined in Ref. 1 and in this Note and those of Ref. 2 are

$$C_{l_p} = (S_F/S_0)(b/d)^2 C_{l_p}^{\mathrm{AD}} \tag{2a}$$

$$C_{l_\delta} = (S_F/S_0)(b/d) C_{l_\delta}^{\mathrm{AD}} \tag{2b}$$

The relationship between the ratios of the stability derivatives becomes

$$C_{l_p}\Big/C_{l_\delta} = (b/d)\Big[C_{l_p}\Big/C_{l_\delta}\Big]^{\mathrm{AD}} \tag{3}$$

The ratio $[C_{l_p}/C_{l_\delta}]^{\mathrm{AD}}$ was obtained from Figs. 6 and 10 of Ref. 2. The results for C_{l_δ} were multiplied by 2.0 to account for the case of four canted fins. The dependence of this ratio on d/b is presented in Fig. 1. It varies between 0.627 for $d/b = 0$ and 1.0 for $d/b = 1.0$.

Eastman's correlation parameter, based on slender-wing analysis by Adams and Dugan, is

$$E = C_{l_p}\Big/\Big[(y_c/d)C_{l_\delta}\Big] = [(b/d)/(y_c/d)]\Big[C_{l_p}\Big/C_{l_\delta}\Big]^{\mathrm{AD}} \tag{4}$$

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Table 1 Comparison of reference quantities

Reference quantity	l_R	S_R	Reduced p
Eastman[1] and present Note	d	S_0	$pd/2v$
Adams and Dugan[2]	b	S_F	$pb/2v$

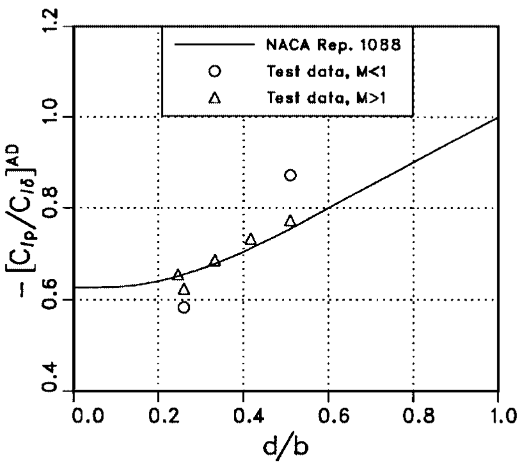

Fig. 1 Ratio of the roll stability derivatives after Adams and Dugan[2] and comparison with test data.

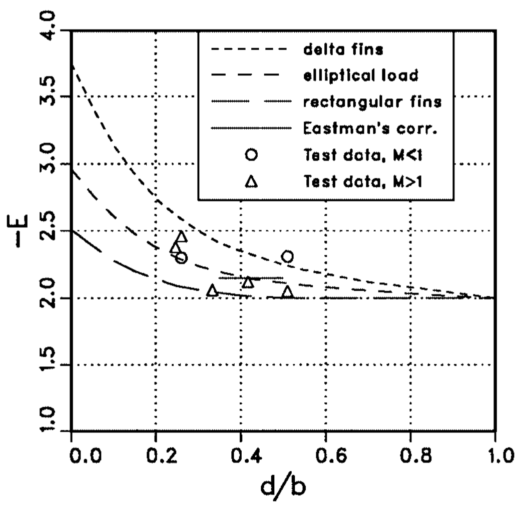

Fig. 2 Predicted Eastman's[1] correlation parameter based on Adams and Dugan[2] analysis and comparison with test data.

This parameter was evaluated for three cases: 1) rectangular fins, for which

$$y_c/d = 0.25 + 0.25(b/d)$$

2) delta fins, for which

$$y_c/d = 0.333 + 0.167(b/d)$$

and 3) elliptical spanwise load distribution, considering the lateral center of pressure rather than the area center:

$$y_c/d = 0.288 + 0.212(b/d)$$

The predicted Eastman correlation parameter is presented in Fig. 2. At $d/b = 0$, the two stability derivatives have no meaning because of the diminishing reference area. Nevertheless, the correlation parameter is finite. The correlation parameter curves decrease as d/b increases and reach a common value of −2.0 at $d/b = 1.0$. The d/b range of the configurations that were analyzed by Eastman was 0.333–0.49. (The folding fin rocket excluded.) The average predicted correlation parameters in this range are $E = -2.35$, −2.15, and −0.201 for delta, elliptical load, and rectangular fins, respectively. The average predicted value for the elliptical load, within this

Table 2 Summary of roll test data

Configuration	d/b	M	C_{l_p}/C_{l_δ}	$-E$	Ref.	Shape
Apache	0.246	2.0–5.0	2.66	2.38	5	Clipped Δ
NASA 80 deg Δ	0.260	0.7–1.0	2.24	2.30	6	Δ
		1.2–1.6	2.40	2.46		
Basic finner	0.333	1.3–3.0	2.06	2.06	7	Square
Naval Ordinance Lab. WAF[a]	0.417	2.5–3.5	1.76	2.12	8	Rectangular
GSRS[b]	0.511	0.7–1.2	1.71	2.31	9	Rectangular
		1.5–2.0	1.51	2.05		

[a]Wraparound fins.
[b]Boeing General Support Rocket System.

range, coincides with the empirical value $E = -2.15$ that was obtained by Eastman, and the largest deviation is only 0.05. It should be noted that the large spread of the three predicted curves is a result of their different shape parameter, y_c/d, rather than of aerodynamic origin.

Sigal[3] analyzed the roll characteristics of several configurations, using the VORLAX code by Miranda et al.[4] It was found that, except for cases of very-low-aspect-ratio fins, the stability derivatives depend on Mach number and generally do not agree with predictions based on the Adams and Dugan[2] analysis. Nevertheless, the ratio of the two subject stability derivatives is almost independent of Mach number and is in good agreement with the predicted ratio. To substantiate this statement, additional test data were compiled. The main geometrical parameters of the test configurations, Mach number ranges and sources,[5–9] are given in Table 2. Most of the data, which are presented in Fig. 2, are scattered about the predicted curve that corresponds to elliptical lateral load distribution and show that indeed Eastman's correlation parameter increases, in absolute value, as b/d decreases.

According to slender-wing theory, the spanwise load distribution is elliptic. Thus, it is expected that for slender fins the lateral center of pressure will only slightly depend on the shape of the planform [provided the similarity parameter $AR\sqrt{(M^2-1)}$ is not large]. Thus, the present correlation does not include a shape parameter. According to Eq. (3), the ratio C_{l_p}/C_{l_δ} increases indefinitely as d/b diminishes. To avoid large variation of the correlation parameter, it was decided to select $(d/b)[C_{l_p}/C_{l_\delta}]$ for comparison with Adams and Dugan analysis.[2] According to Eq. (3),

$$(d/b)\left[C_{l_p}/C_{l_\delta}\right] = \left[C_{l_p}/C_{l_\delta}\right]^{\mathrm{AD}} \tag{5}$$

A comparison between the test data of Table 2 and analysis of Adams and Dugan is also shown in Fig. 1. The supersonic data are on or very close to the analytical curve, and the subsonic data are near it. The ratio of the two roll stability derivatives depends on the d/b ratio, as expected from the analysis by Adams and Dugan.

Summary

The results of Adams and Dugan's analysis[2] of the roll stability derivatives for cruciform wings and body configurations were used to predict Eastman's correlation[1] of the ratio of these stability derivatives. It was found that the predicted Eastman correlation parameter depends on fin planform and decreases, in absolute value, as d/b increases. For $d/b = 0.333$–0.49, which cover the configurations studied by Eastman, the average predicted value of the correlation parameter for elliptical spanwise load distribution is exactly $E = -2.15$, which was empirically obtained by him.

Conclusion

The present correlation of the ratio of the roll stability derivatives is in very good agreement with that calculated by Adams and Dugan. This finding confirms the use of their analysis for the prediction of the ratio of the roll stability derivatives for configurations employing slender fins.

References

[1]Eastman, D. W., "Roll Damping of Cruciform-Tailed Missiles," *Journal of Spacecraft and Rockets*, Vol. 33, No. 1, 1986, pp. 119, 120.

[2]Adams, G. J., and Dugan, D. W., "Theoretical Damping in Roll and Rolling Moment Due to Differential Wing Incidence for Slender Cruciform Wings and Wing–Body Combinations," NACA Rept. 1088, 1952.

[3]Sigal, A., "Analysis of the Roll Moments of Finned Missiles," AIAA Paper 97-3494, Aug. 1997.

[4]Miranda, L. R., Elliot, R. D., and Baker, W. M., "A Generalized Vortex Lattice Method for Subsonic and Supersonic Flow," NASA CR-2865, Dec. 1977.

[5]Uselton, J. C., and Carman, J. B., "Wind Tunnel Tests of the Apache Sounding Rocket with Various Spin Rates at Supersonic Speeds," Arnold Engineering Development Center, Rept. AEDC-TR-69-57, ARO, Inc., Arnold AFB, TN, May 1969.

[6]Schult, E. D., "Free Flight Investigation at Mach Numbers Between 0.5 and 1.7 of the Zero-Lift Rolling Effectiveness and Drag of Various Surfaces, Spoilers, and Jet Controls on a 80° Delta-Wing Missile," NASA TN-D-205, Feb. 1960.

[7]Nicolaides, J. D., and Bolz, R. E., "On the Pure Rolling Motion of Winged and/or Finned Missiles in Varying Supersonic Flight," *Journal of the Aeronautical Sciences*, Vol. 20, No. 3, 1953, pp. 160–168.

[8]Regan, F. J., "Supersonic Magnus Measurements of the 10-Caliber Army-Navy Spinner Projectile with Wrap-Around Fins," U.S. Naval Ordnance Lab., Rept. NOLTR-70-211, White Oak, MD, Oct. 1970.

[9]Mikhail, A. G., "Roll Damping for Projectiles Including Wraparound, Offset, and Arbitrary Number of Fins," *Journal of Spacecraft and Rockets*, Vol. 32, No. 6, 1995, pp. 929–937.

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